

A New SLNR-based Linear Precoding for Downlink Multi-User Multi-Stream MIMO Systems

Peng Cheng, Meixia Tao and Wenjun Zhang

Abstract

Signal-to-leakage-and-noise ratio (SLNR) is a promising criterion for linear precoder design in multi-user (MU) multiple-input multiple-output (MIMO) systems. It decouples the precoder design problem and makes closed-form solution available. In this letter, we present a new linear precoding scheme by slightly relaxing the SLNR maximization for MU-MIMO systems with multiple data streams per user. The precoding matrices are obtained by a general form of simultaneous diagonalization of two Hermitian matrices. The new scheme reduces the gap between the per-stream effective channel gains, an inherent limitation in the original SLNR precoding scheme. Simulation results demonstrate that the proposed precoding achieves considerable gains in error performance over the original one for multi-stream transmission while maintaining almost the same achievable sum-rate.

Index Terms

Signal-to-leakage-and-noise ratio (SLNR), linear precoding, multi-user MIMO.

I. INTRODUCTION

The significance of a downlink multi-user multiple-input-multiple-output (MU-MIMO) system is to allow a base station (BS) to communicate with several co-channel mobile stations (MS) simultaneously and thereby considerably increase the system throughput. To utilize the benefit, it is essential to suppress co-channel interference (CCI). Among many CCI suppression schemes,

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linear precoding gains the popularity because of its simplicity for implementation and good performance. To design the optimal linear MU-MIMO precoding scheme, it is often desirable to maximize the output signal-to-interference-plus-noise ratio (SINR) for each user. However, this problem is known to be challenging due to its coupled nature and no closed-form solution is available yet. A more tractable but suboptimal design is to enforce a zero-CCI requirement for each user, such as block diagonalization (BD) [1] and coordinated beamforming (CB) [2].

In [3], the authors propose a so-called signal-to-leakage-and-noise ratio (SLNR) as the optimization metric for linear precoder design. This metric transforms a coupled optimization problem into a completely decoupled one, for which a closed-form solution is available. Unlike the BD approach, it does not impose a restriction on the number of transmit antennas at the BS. Moreover, it is applicable for any number of users and data streams in contrast to CB scheme. Specifically, the SLNR based linear precoding weights in [3] are obtained by the generalized eigenvalue decomposition (GED) of the channel covariance matrix and the leakage channel-plus-noise covariance matrix of each user. However, a drawback of such GED based precoding scheme is that, when each user has multiple data streams, the effective channel gain for each stream can be severely unbalanced. If power control or adaptive modulation and coding cannot be applied, the overall error performance of each user will suffer significant loss.

In this letter, we present a new linear precoding scheme based on the SLNR criterion for a downlink MU-MIMO system with multiple data streams per user. The design goal is to reduce the margin between the effective SINRs of multiple data streams. To do this, we introduce a slight relaxation for pursuing SLNR maximization (Note that maximizing SLNR at the transmitter side does not necessarily lead to output SINR maximization at each receiver). Thereby, we obtain a general form of simultaneous diagonalization of two covariance matrices linked to the user's channel and leakage-plus-noise. Based on that, the new precoding matrices are then obtained. We also present a simple and low-complexity algorithm to compute the precoding matrix for each user. Simulation results confirm that, compared with the original scheme, our scheme demonstrates sizable performance gains in error rate performance for multi-stream transmission while maintaining almost the same sum-rate performance.

Notations: $\mathbb{E}(\cdot)$, $\text{Tr}(\cdot)$, $(\cdot)^{-1}$, and $(\cdot)^H$ denote expectation, trace, inverse, and conjugate transpose, respectively. $\|\cdot\|_F$ represents the Frobenius norm. \mathbf{I}_N is the $N \times N$ identity matrix. $\text{diag}(a_1, \dots, a_N)$ is the diagonal matrix with element a_n on the n -th diagonal. Besides, $\mathbb{C}^{M \times N}$

represents the set of $M \times N$ matrices in complex field.

II. SYSTEM MODEL

We consider a downlink MU-MIMO system with N transmit antennas and M receive antennas at each of the K active users. Let $\mathbf{H}_k \in \mathbb{C}^{M \times N}$ denotes the channel from the BS to the MS k and $\bar{\mathbf{H}}_k = [\mathbf{H}_1^H, \dots, \mathbf{H}_{k-1}^H, \mathbf{H}_{k+1}^H, \dots, \mathbf{H}_K^H]^H \in \mathbb{C}^{(K-1)M \times N}$ represent the corresponding concatenated leakage channel. A spatially uncorrelated flat Rayleigh fading channel is assumed. The elements of \mathbf{H}_k are modeled as independent and identically distributed complex Gaussian variables with zero-mean and unit-variance. In addition, we assume \mathbf{H}_k , and also $\bar{\mathbf{H}}_k$, have full rank with probability one. For a specific vector time, the transmitted vector symbol of user k is denoted as $\mathbf{s}_k \in \mathbb{C}^{L \times 1}$, where $L (\leq M)$ is the number of data streams supported for user k and is assumed equal for all the users for simplicity. The vector symbol satisfies the power constraint $\mathbb{E}(\mathbf{s}_k \mathbf{s}_k^H) = \mathbf{I}_L$. Before entering into the MIMO channel, the vector \mathbf{s}_k is pre-multiplied by a precoding matrix $\mathbf{F}_k \in \mathbb{C}^{N \times L}$. Here, power allocation and rate adaptation among data streams can be applied. However, the signal design or feedback support may be relatively complex and thus we resort to precoding design only in this work. Then, for a given user k , the received signal vector can be written as

$$\mathbf{r}_k = \mathbf{H}_k \mathbf{F}_k \mathbf{s}_k + \mathbf{H}_k \sum_{i=1, i \neq k}^K \mathbf{F}_i \mathbf{s}_i + \mathbf{n}_k \quad (1)$$

in which the second term represents CCI and the third term is the additive white Gaussian noise with $\mathbb{E}(\mathbf{n}_k \mathbf{n}_k^H) = \sigma^2 \mathbf{I}_M$.

We review the original SLNR based precoding scheme in [3]. Recall that the SLNR is defined as the ratio of received signal power at the desired MS to received signal power at the other terminals (the leakage) plus noise power without considering receive matrices, given by

$$\text{SLNR}_k = \frac{\text{Tr}(\mathbf{F}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{F}_k)}{\text{Tr}(\mathbf{F}_k^H (M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k) \mathbf{F}_k)}, \quad (2)$$

for $k = 1, \dots, K$. According to the SLNR criterion, the precoding matrix \mathbf{F}_k is designed based on the following metric

$$\mathbf{F}_k^{\text{opt}} = \arg \max_{\mathbf{F}_k \in \mathbb{C}^{N \times L}} \text{SLNR}_k \quad (3)$$

with $\text{Tr}(\mathbf{F}_k \mathbf{F}_k^H) = L$ for power limitation. Since $\mathbf{H}_k^H \mathbf{H}_k$ is Hermitian and positive semidefinite (HPSD) and $M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k$ is Hermitian and positive definite (HPD), by generalized

eigenvalue decomposition, there exists an invertible matrix $\mathbf{T}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{T}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{T}_k = \Lambda_k = \text{diag}(\lambda_1, \dots, \lambda_N) \quad (4)$$

$$\mathbf{T}_k^H (M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k) \mathbf{T}_k = \mathbf{I}_N \quad (5)$$

with $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N \geq 0$. Here, the columns of \mathbf{T}_k and the diagonal entries of Λ_k are the generalized eigenvectors and eigenvalues of the pair $\{\mathbf{H}_k^H \mathbf{H}_k, M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k\}$, respectively. It is then shown in [3] that the optimal precoder which is able to maximize the objective function (3) can be obtained by extracting the leading L columns of \mathbf{T}_k as

$$\mathbf{F}_k^{\text{opt}} = \rho \mathbf{T}_k [\mathbf{I}_L; \mathbf{0}], \quad (6)$$

where ρ is a scaling factor so that $\text{Tr}(\mathbf{F}_k \mathbf{F}_k^H) = L$. The resulting maximum SLNR value is given by $\text{SLNR}_k^{\max} = \sum_{i=1}^L \lambda_i / L$. Along with the realization of the precoder, the matched-filter type receive matrix, denoted as $\mathbf{G}_k = (\mathbf{H}_k \mathbf{F}_k)^H$, is applied at each user receiver, resulting in inter-stream interference free. Note that better performance could be achieved if a multi-user MMSE type receiver is adopted. In this letter, we still adopt MF-type detector at the receiver as in [3] for implementation simplicity and analytical convenience.

A drawback of such GED based precoding scheme is that, when $L \geq 2$, the effective channel gain for each stream can be severely unbalanced as shall be illustrated in Section III-C. It is known that the overall performance of a user with multiple streams is dominated by the stream with the worst channel condition. Hence, such channel imbalance would lead to poor overall error performance for a user. In the next section, we allow a slight relaxation on the SLNR maximization, which provides additional degrees of freedom to design a new precoding scheme so as to overcome this drawback.

III. PROPOSED PRECODING SCHEME

A. Design Principle by Matrix Theory

The expressions in (4) and (5) by the GED approach motivate us to find a more general form of simultaneous diagonalization of two matrices. Before introducing our results in Proposition 1, we review the following Lemma [4, Ch. 4, 4.5.8]:

Lemma 1: Let $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{n \times n}$ be Hermitian. There is a non-singular matrix $\mathbf{S} \in \mathbb{C}^{n \times n}$ such that $\mathbf{S}^H \mathbf{A} \mathbf{S} = \mathbf{B}$ if and only if \mathbf{A} and \mathbf{B} have the same inertia, that is, have the same number of positive, negative, and zero eigenvalues.

Proposition 1: For the pair of matrices $\{\mathbf{H}_k^H \mathbf{H}_k, M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k\}$, there is a non-singular matrix $\mathbf{P}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{P}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{P}_k = \Theta_k \quad (7)$$

$$\mathbf{P}_k^H (M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k) \mathbf{P}_k = \Omega_k \quad (8)$$

in which $\Theta_k = \text{diag}(\theta_1, \theta_2, \dots, \theta_N)$ and $\Omega_k = \text{diag}(\omega_1, \omega_2, \dots, \omega_N)$ with the entries satisfying $1 > \theta_1 \geq \dots \geq \theta_M > 0$, $\theta_{M+1} = \dots = \theta_N = 0$ and $0 < \omega_1 \leq \dots \leq \omega_M < 1$, $\omega_{M+1} = \dots = \omega_N = 1$ as well as $\theta_i + \omega_i = 1$ for $i = 1, 2, \dots, N$.

Proof: Denote $\mathbf{A}_k = \mathbf{H}_k^H \mathbf{H}_k$, $\mathbf{B}_k = M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k$ and $\mathbf{C}_k = \mathbf{A}_k + \mathbf{B}_k$. Let the eigenvalues $\lambda_i(\mathbf{A}_k)$, $\lambda_i(\mathbf{B}_k)$ and $\lambda_i(\mathbf{C}_k)$, $i = 1, 2, \dots, N$, be arranged in increasing order. Since \mathbf{A}_k is HPSD and \mathbf{B}_k is HPD, namely, $\lambda_i(\mathbf{A}_k) \geq 0$ and $\lambda_i(\mathbf{B}_k) > 0$, then by [4, 4.3.1], we have $\lambda_i(\mathbf{C}_k) \geq \lambda_i(\mathbf{A}_k) + \lambda_1(\mathbf{B}_k) > 0$, $\forall i$. This implies that \mathbf{C}_k is HPD. Then, by the matrix theory in [4, 4.5.8, Exercise], there must be a non-singular matrix $\mathbf{Q}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{Q}_k^H \mathbf{C}_k \mathbf{Q}_k = \mathbf{Q}_k^H (\mathbf{A}_k + \mathbf{B}_k) \mathbf{Q}_k = \mathbf{I}_N. \quad (9)$$

Further, denote $\mathbf{A}'_k = \mathbf{Q}_k^H \mathbf{A}_k \mathbf{Q}_k$ and $\mathbf{B}'_k = \mathbf{Q}_k^H \mathbf{B}_k \mathbf{Q}_k$. By Lemma 1, it can be shown that \mathbf{A}'_k and \mathbf{B}'_k have the same inertia with \mathbf{A}_k and \mathbf{B}_k , respectively. Thus, \mathbf{A}'_k is HPSD and \mathbf{B}'_k is HPD. Now, by using [4, 4.3.1] again, it is easy to show that $1 > \lambda_i(\mathbf{A}'_k) \geq 0$ and $1 \geq \lambda_i(\mathbf{B}'_k) > 0$. Next, according to the eigen-decomposition (ED) of a Hermitian matrix [4], there must be a unitary matrix $\mathbf{U}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{U}_k^H \mathbf{A}'_k \mathbf{U}_k = \text{diag}(\lambda_1(\mathbf{A}'_k), \dots, \lambda_N(\mathbf{A}'_k)). \quad (10)$$

Applying \mathbf{U}_k in both sides of (9), we obtain

$$\mathbf{U}_k^H (\mathbf{A}'_k + \mathbf{B}'_k) \mathbf{U}_k = \mathbf{I}_N. \quad (11)$$

Hence, observing (10) and (11), we find that it is necessary for $\mathbf{U}_k^H \mathbf{B}'_k \mathbf{U}_k$ to satisfy $\mathbf{U}_k^H \mathbf{B}'_k \mathbf{U}_k = \text{diag}((1 - \lambda_1(\mathbf{A}'_k)), \dots, (1 - \lambda_N(\mathbf{A}'_k)))$. Clearly, as \mathbf{U}_k is unitary, then $\{1 - \lambda_i(\mathbf{A}'_k)\}_{i=1}^N$ must be the eigenvalues of \mathbf{B}'_k . To this end, we define $\mathbf{P}_k = \mathbf{Q}_k \mathbf{U}_k$. Since $\text{rank}(\mathbf{H}_k^H \mathbf{H}_k) = \text{rank}(\mathbf{H}_k) = M$ and the rank is unchanged upon left or right multiplication by a nonsingular matrix, then we arrive at the results in (7) and (8). \blacksquare

Algorithm 1 The specific design of precoder \mathbf{F}'_k for user k

Input: $\mathbf{A}_k = \mathbf{H}_k^H \mathbf{H}_k$, and $\mathbf{C}_k = (\mathbf{H}_k^H \mathbf{H}_k + M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k)$

- 1) Compute Cholesky decomposition on \mathbf{C}_k , as $\mathbf{C}_k = \mathbf{G}_k \mathbf{G}_k^H$, where $\mathbf{G}_k \in \mathbb{C}^{N \times N}$ is a lower triangular matrix with positive diagonal entries. Then, \mathbf{G}_k^{-1} can be easily obtained and we have $(\mathbf{G}_k^{-1})^H = \mathbf{Q}_k$ in (9).
- 2) Compute $\mathbf{A}'_k = \mathbf{Q}_k^H \mathbf{A}_k \mathbf{Q}_k$, then compute ED on \mathbf{A}'_k as $\mathbf{A}'_k \mathbf{U}_k = \mathbf{U}_k \Lambda_k$. Note \mathbf{U}_k must be unitary and it can be also obtained by computing the left singular matrix of \mathbf{A}'_k in terms of SVD.
- 3) Compute $\mathbf{P}_k = \mathbf{Q}_k \mathbf{U}_k$.

Output: $\mathbf{F}'_k = \gamma \mathbf{P}_k (\mathbf{I}_L; \mathbf{0})$.

B. Precoder Design

The simultaneous diagonalization in general form stated in Proposition 1 draws a significant distinction from the original GED based deduction in (4) and (5). This allows us to design a new precoding scheme. In specific, the proposed precoder \mathbf{F}'_k and matched decoder \mathbf{G}'_k can be designed as

$$\mathbf{F}'_k = \gamma \mathbf{P}_k [\mathbf{I}_L; \mathbf{0}], \quad \mathbf{G}'_k = (\mathbf{H}_k \mathbf{F}'_k)^H \quad (12)$$

in which γ is a normalization factor so that $\text{Tr}(\mathbf{F}'_k \mathbf{F}'_k^H) = L$. It is clear that $\mathbf{G}'_k \mathbf{H}_k \mathbf{F}_k$ amounts to a certain diagonal matrix, also resulting in inter-stream-interference free.

The remaining problem is how to compute a specific precoder \mathbf{F}'_k for each user. Based on our proof of Proposition 1, we present a closed-form expression using a simple and low-complex algorithm, outlined in Algorithm 1. In the next subsection, we reveal the superiority of the proposed precoding scheme through per-stream SINR discussion.

C. Performance Discussion

Firstly, continuing to use the same symbols \mathbf{A}_k and \mathbf{B}_k as in the proof of Proposition 1, we can show that $\mathbf{A}_k \mathbf{t}_{k_i} = \lambda_i \mathbf{B}_k \mathbf{t}_{k_i}$ and $\mathbf{A}_k \mathbf{p}_{k_i} = (\theta_i/\omega_i) \mathbf{B}_k \mathbf{p}_{k_i}$ from (5) and (8), in which \mathbf{t}_{k_i} and \mathbf{p}_{k_i} correspond to the i -th column of \mathbf{T}_k and \mathbf{P}_k , respectively. Here, both λ_i and θ_i/ω_i must be the generalized eigenvalues of the pair $\{\mathbf{A}_k, \mathbf{B}_k\}$. It is then easy to see that

$$\lambda_j = \theta_j/\omega_j, \quad j = 1, 2, \dots, N \quad (13)$$

with $\{\lambda_j\}_{j=1}^N$ and $\{\theta_j\}_{j=1}^N$ being sorted in descending order while $\{\omega_j\}_{j=1}^N$ sorted in ascending order. Now we have $\text{SLNR}_k = \left(\sum_{l=1}^L \theta_l \right) / \left(\sum_{l=1}^L (1 - \theta_l) \right)$, which is slightly smaller than

SLNR_k^{\max} given in Section II.

On the other hand, the ultimate performance is decided by post-SINR. Clearly, the decoded signal should take the form

$$\hat{s}_k = \mathbf{G}'_k \mathbf{H}_k \mathbf{F}_k s_k + \mathbf{G}'_k \left(\mathbf{H}_k \sum_{i=1, i \neq k}^K \mathbf{F}_i s_i + \mathbf{n}_k \right). \quad (14)$$

Thanks to diagonal form in (7) and (8), the covariance matrix of noise vector is given by $\mathbb{E}(\mathbf{G}'_k \mathbf{n}_k \mathbf{n}_k^H \mathbf{G}'_k^H) = \sigma^2 \mathbf{I}_L \mathbb{E}(\mathbf{G}'_k \mathbf{G}'_k^H) = \gamma^2 \sigma^2 \mathbf{I}_L \text{diag}(\theta_1, \dots, \theta_L)$. Furthermore, it can be verified through numerical results (difficult via theoretical analysis though) that the residual CCI is much smaller than the noise power at high SNR. As such, the SINR on the l -th stream, η'_l can be approximately calculated as $\eta'_l = (\gamma^4 \theta_l^2) / (\gamma^2 \sigma^2 \theta_l) = \gamma^2 \theta_l / \sigma^2$. Then, for any two streams l and m with $l > m$, the margin of $\Delta'_{l,m}$ between η'_l and η'_m in terms of decibel (dB) can be expressed as

$$\Delta'_{l,m} = 10 \log_{10} (\eta'_l / \eta'_m) = 10 \log_{10} (\theta_l / \theta_m). \quad (15)$$

Following the same analysis, the margin of $\Delta_{l,m}$ for the original scheme can be analogously calculated as

$$\Delta_{l,m} = 10 \log_{10} (\eta_l / \eta_m) = 10 \log_{10} (\lambda_l / \lambda_m). \quad (16)$$

According to (13), we have $\lambda_l / \lambda_m = (\theta_l \omega_m) / (\theta_m \omega_l)$. Further, we have that $\omega_m > \omega_l$ for $l > m$ by definition. It then ensures that the following inequality holds:

$$\Delta'_{l,m} < \Delta_{l,m}. \quad (17)$$

This explicitly shows that the SINR margin between any two streams decreases by applying the proposed scheme. In other words, the effective channel gains between the multiple streams are now less unbalanced. Its effectiveness will be further examined by simulation in the next section.

IV. SIMULATION RESULTS

Fig. 1 compares the simulated bit error rate (BER) per user in a MU-MIMO system with different system configurations. Here, P denotes the proposed precoding scheme and O denotes the original scheme in [3]. QPSK modulation with Gray mapping is employed and the BER curves are plotted versus the transmit SNR (L/σ^2). It is seen that the proposed scheme and the original scheme for single-stream case ($L = 1$) achieve the same BER performance. For multiple streams ($L = 2$ and 3), the former outperforms the latter with sizeable gains. In specific, a gain

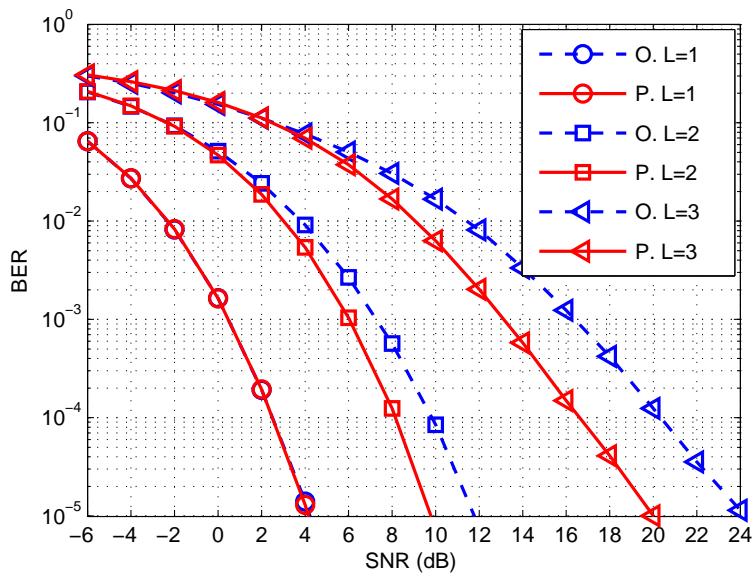


Fig. 1. Uncoded BER of a MU-MIMO system with $N = 8$ transmit antennas at the BS and $K = 2$ users each with $M = 3$ receive antennas.

of around 2 dB and 4 dB can be achieved at $\text{BER}=10^{-4}$ for streams of $L = 2$ and $L = 3$, respectively. We also carried out the achievable sum-rate comparison. It is found that our scheme is almost the same as the original one. The results are omitted due to page limit.

The above simulation results verify the effectiveness of the proposed precoding scheme over the original SLNR based scheme when there are multiple data streams for each user.

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of the channel covariance matrix and the leakage channel-plus-noise covariance matrix of each user. However, a drawback of such GED based precoding scheme is that, when each user has multiple data streams, the effective channel gain for each stream can be severely unbalanced. If power control or adaptive modulation and coding cannot be applied, the overall error performance of each user will suffer significant loss.

In this letter, we present a new linear precoding scheme based on the SLNR criterion for a downlink MU-MIMO system with multiple data streams per user. The design goal is to reduce the margin between the effective SINRs of multiple data streams. To do this, we introduce a slight relaxation for pursuing SLNR maximization (Note that maximizing SLNR at the transmitter side does not necessarily lead to output SINR maximization at each receiver). Thereby, we obtain a general form of simultaneous diagonalization of two covariance matrices linked to the user's channel and leakage-plus-noise. Based on that, the new precoding matrices are then obtained. We also present a simple and low-complexity algorithm to compute the precoding matrix for each user. Simulation results confirm that, compared with the original scheme, our scheme demonstrates sizable performance gains in error rate performance for multi-stream transmission while maintaining almost the same sum-rate performance.

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vector \mathbf{s}_k is pre-multiplied by a precoding matrix $\mathbf{F}_k \in \mathbb{C}^{N \times L}$. Here, power allocation and rate adaptation among data streams can be applied. However, the signal design or feedback support may be relatively complex and thus we resort to precoding design only in this work. Then, for a given user k , the received signal vector can be written as

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in which the second term represents CCI and the third term is the additive white Gaussian noise with $\mathbb{E}(\mathbf{n}_k \mathbf{n}_k^H) = \sigma^2 \mathbf{I}_M$.

We review the original SLNR based precoding scheme in [3]. Recall that the SLNR is defined as the ratio of received signal power at the desired MS to received signal power at the other terminals (the leakage) plus noise power without considering receive matrices, given by

$$\text{SLNR}_k = \frac{\text{Tr}(\mathbf{F}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{F}_k)}{\text{Tr}(\mathbf{F}_k^H (M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k) \mathbf{F}_k)}, \quad (2)$$

for $k = 1, \dots, K$. According to the SLNR criterion, the precoding matrix \mathbf{F}_k is designed based on the following metric

$$\mathbf{F}_k^{\text{opt}} = \arg \max_{\mathbf{F}_k \in \mathbb{C}^{N \times L}} \text{SLNR}_k \quad (3)$$

with $\text{Tr}(\mathbf{F}_k \mathbf{F}_k^H) = L$ for power limitation. Since $\mathbf{H}_k^H \mathbf{H}_k$ is Hermitian and positive semidefinite (HPSD) and $M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k$ is Hermitian and positive definite (HPD), by generalized eigenvalue decomposition, there exists an invertible matrix $\mathbf{T}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{T}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{T}_k = \mathbf{\Lambda}_k = \text{diag}(\lambda_1, \dots, \lambda_N) \quad (4)$$

$$\mathbf{T}_k^H (M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k) \mathbf{T}_k = \mathbf{I}_N \quad (5)$$

with $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N \geq 0$. Here, the columns of \mathbf{T}_k and the diagonal entries of $\mathbf{\Lambda}_k$ are the generalized eigenvectors and eigenvalues of the pair $\{\mathbf{H}_k^H \mathbf{H}_k, M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k\}$, respectively. It is then shown in [3] that the optimal precoder which is able to maximize the objective function (3) can be obtained by extracting the leading L columns of \mathbf{T}_k as

$$\mathbf{F}_k^{\text{opt}} = \rho \mathbf{T}_k [\mathbf{I}_L; \mathbf{0}], \quad (6)$$

where ρ is a scaling factor so that $\text{Tr}(\mathbf{F}_k \mathbf{F}_k^H) = L$. The resulting maximum SLNR value is given by $\text{SLNR}_k^{\text{max}} = \sum_{i=1}^L \lambda_i / L$. Along with the realization of the precoder, the matched-filter type receive matrix, denoted as $\mathbf{G}_k = (\mathbf{H}_k \mathbf{F}_k)^H$, is applied at each user receiver, resulting in inter-stream interference free. Note that better performance could be achieved if a multi-user MMSE type receiver is adopted. In this letter, we still adopt MF-type detector at the receiver as in [3] for implementation simplicity and analytical convenience.

A drawback of such GED based precoding scheme is that, when $L \geq 2$, the effective channel gain for each stream can be severely unbalanced as shall be illustrated in Section III-C. It is known that the overall performance of a user with multiple streams is dominated by the stream with the worst channel condition. Hence, such channel imbalance would lead to poor overall error performance for a user. In the next section, we allow a slight relaxation on the SLNR maximization, which provides additional degrees of freedom to design a new precoding scheme so as to overcome this drawback.

III. PROPOSED PRECODING SCHEME

A. Design Principle by Matrix Theory

The expressions in (4) and (5) by the GED approach motivate us to find a more general form of simultaneous diagonalization of two matrices. Before introducing our results in Proposition 1, we review the following Lemma [4, Ch. 4, 4.5.8]:

Lemma 1: Let $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{n \times n}$ be Hermitian. There is a non-singular matrix $\mathbf{S} \in \mathbb{C}^{n \times n}$ such that $\mathbf{S}^H \mathbf{A} \mathbf{S} = \mathbf{B}$ if and only if \mathbf{A} and \mathbf{B} have the same inertia, that is, have the same number of positive, negative, and zero eigenvalues.

Proposition 1: For the pair of matrices $\{\mathbf{H}_k^H \mathbf{H}_k, M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k\}$, there is a non-singular matrix $\mathbf{P}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{P}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{P}_k = \mathbf{\Theta}_k \quad (7)$$

$$\mathbf{P}_k^H (M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k) \mathbf{P}_k = \mathbf{\Omega}_k \quad (8)$$

in which $\mathbf{\Theta}_k = \text{diag}(\theta_1, \theta_2, \dots, \theta_N)$ and $\mathbf{\Omega}_k = \text{diag}(\omega_1, \omega_2, \dots, \omega_N)$ with the entries satisfying $1 > \theta_1 \geq \dots \geq \theta_M > 0$, $\theta_{M+1} = \dots = \theta_N = 0$ and $0 < \omega_1 \leq \dots \leq \omega_M < 1$, $\omega_{M+1} = \dots = \omega_N = 1$ as well as $\theta_i + \omega_i = 1$ for $i = 1, 2, \dots, N$.

Proof: Denote $\mathbf{A}_k = \mathbf{H}_k^H \mathbf{H}_k$, $\mathbf{B}_k = M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k$ and $\mathbf{C}_k = \mathbf{A}_k + \mathbf{B}_k$. Let the eigenvalues $\lambda_i(\mathbf{A}_k)$, $\lambda_i(\mathbf{B}_k)$ and $\lambda_i(\mathbf{C}_k)$, $i = 1, 2, \dots, N$, be arranged in increasing order. Since \mathbf{A}_k is HPSD and \mathbf{B}_k is HPD, namely, $\lambda_i(\mathbf{A}_k) \geq 0$ and $\lambda_i(\mathbf{B}_k) > 0$, then by [4, 4.3.1], we have $\lambda_i(\mathbf{C}_k) \geq \lambda_i(\mathbf{A}_k) + \lambda_1(\mathbf{B}_k) > 0$, $\forall i$. This implies that \mathbf{C}_k is HPD. Then, by the matrix theory in [4, 4.5.8, Exercise], there must be a non-singular matrix $\mathbf{Q}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{Q}_k^H \mathbf{C}_k \mathbf{Q}_k = \mathbf{Q}_k^H (\mathbf{A}_k + \mathbf{B}_k) \mathbf{Q}_k = \mathbf{I}_N. \quad (9)$$

Further, denote $\mathbf{A}'_k = \mathbf{Q}_k^H \mathbf{A}_k \mathbf{Q}_k$ and $\mathbf{B}'_k = \mathbf{Q}_k^H \mathbf{B}_k \mathbf{Q}_k$. By Lemma 1, it can be shown that \mathbf{A}'_k and \mathbf{B}'_k have the same inertia with \mathbf{A}_k and \mathbf{B}_k , respectively. Thus, \mathbf{A}'_k is HPSD and \mathbf{B}'_k is HPD. Now, by using [4, 4.3.1] again, it is easy to show that $1 > \lambda_i(\mathbf{A}'_k) \geq 0$ and $1 \geq \lambda_i(\mathbf{B}'_k) > 0$. Next, according to the eigen-decomposition (ED) of a Hermitian matrix [4], there must be a unitary matrix $\mathbf{U}_k \in \mathbb{C}^{N \times N}$ such that

$$\mathbf{U}_k^H \mathbf{A}'_k \mathbf{U}_k = \text{diag}(\lambda_1(\mathbf{A}'_k), \dots, \lambda_N(\mathbf{A}'_k)). \quad (10)$$

Applying \mathbf{U}_k in both sides of (9), we obtain

$$\mathbf{U}_k^H (\mathbf{A}'_k + \mathbf{B}'_k) \mathbf{U}_k = \mathbf{I}_N. \quad (11)$$

Hence, observing (10) and (11), we find that it is necessary for $\mathbf{U}_k^H \mathbf{B}'_k \mathbf{U}_k$ to satisfy $\mathbf{U}_k^H \mathbf{B}'_k \mathbf{U}_k = \text{diag}((1 - \lambda_1(\mathbf{A}'_k)), \dots, (1 - \lambda_N(\mathbf{A}'_k)))$. Clearly, as \mathbf{U}_k is unitary, then $\{1 - \lambda_i(\mathbf{A}'_k)\}_{i=1}^N$ must be the eigenvalues of \mathbf{B}'_k . To this end, we define $\mathbf{P}_k = \mathbf{Q}_k \mathbf{U}_k$. Since $\text{rank}(\mathbf{H}_k^H \mathbf{H}_k) = \text{rank}(\mathbf{H}_k) = M$ and the rank is unchanged upon left or right multiplication by a nonsingular matrix, then we arrive at the results in (7) and (8). ■

B. Precoder Design

The simultaneous diagonalization in general form stated in Proposition 1 draws a significant distinction from the original GED based deduction in (4) and (5). This allows us to design

Algorithm 1 The specific design of precoder \mathbf{F}'_k for user k

Input: $\mathbf{A}_k = \mathbf{H}_k^H \mathbf{H}_k$, and $\mathbf{C}_k = (\mathbf{H}_k^H \mathbf{H}_k + M/L\sigma^2 \mathbf{I} + \bar{\mathbf{H}}_k^H \bar{\mathbf{H}}_k)$
 1) Compute Cholesky decomposition on \mathbf{C}_k , as $\mathbf{C}_k = \mathbf{G}_k \mathbf{G}_k^H$, where $\mathbf{G}_k \in \mathbb{C}^{N \times N}$ is a lower triangular matrix with positive diagonal entries. Then, \mathbf{G}_k^{-1} can be easily obtained and we have $(\mathbf{G}_k^{-1})^H = \mathbf{Q}_k$ in (9).
 2) Compute $\mathbf{A}'_k = \mathbf{Q}_k^H \mathbf{A}_k \mathbf{Q}_k$, then compute ED on \mathbf{A}'_k as $\mathbf{A}'_k \mathbf{U}_k = \mathbf{U}_k \mathbf{A}_k$. Note \mathbf{U}_k must be unitary and it can be also obtained by computing the left singular matrix of \mathbf{A}'_k in terms of SVD.
 3) Compute $\mathbf{P}_k = \mathbf{Q}_k \mathbf{U}_k$.
 Output: $\mathbf{F}'_k = \gamma \mathbf{P}_k (\mathbf{I}_L; \mathbf{0})$.

a new precoding scheme. In specific, the proposed precoder \mathbf{F}'_k and matched decoder \mathbf{G}'_k can be designed as

$$\mathbf{F}'_k = \gamma \mathbf{P}_k [\mathbf{I}_L; \mathbf{0}], \quad \mathbf{G}'_k = (\mathbf{H}_k \mathbf{F}'_k)^H \quad (12)$$

in which γ is a normalization factor so that $\text{Tr}(\mathbf{F}'_k \mathbf{F}'_k^H) = L$. It is clear that $\mathbf{G}'_k \mathbf{H}_k \mathbf{F}'_k$ amounts to a certain diagonal matrix, also resulting in inter-stream-interference free.

The remaining problem is how to compute a specific precoder \mathbf{F}'_k for each user. Based on our proof of Proposition 1, we present a closed-form expression using a simple and low-complex algorithm, outlined in Algorithm 1. In the next subsection, we reveal the superiority of the proposed precoding scheme through per-stream SINR discussion.

C. Performance Discussion

Firstly, continuing to use the same symbols \mathbf{A}_k and \mathbf{B}_k as in the proof of Proposition 1, we can show that $\mathbf{A}_k \mathbf{t}_{k_i} = \lambda_i \mathbf{B}_k \mathbf{t}_{k_i}$ and $\mathbf{A}_k \mathbf{p}_{k_i} = (\theta_i/\omega_i) \mathbf{B}_k \mathbf{p}_{k_i}$ from (5) and (8), in which \mathbf{t}_{k_i} and \mathbf{p}_{k_i} correspond to the i -th column of \mathbf{T}_k and \mathbf{P}_k , respectively. Here, both λ_i and θ_i/ω_i must be the generalized eigenvalues of the pair $\{\mathbf{A}_k, \mathbf{B}_k\}$. It is then easy to see that

$$\lambda_j = \theta_j/\omega_j, \quad j = 1, 2, \dots, N \quad (13)$$

with $\{\lambda_j\}_{j=1}^N$ and $\{\theta_j\}_{j=1}^N$ being sorted in descending order while $\{\omega_j\}_{j=1}^N$ sorted in ascending order. Now we have $\text{SLNR}_k = \left(\sum_{l=1}^L \theta_l \right) / \left(\sum_{l=1}^L (1 - \theta_l) \right)$, which is slightly smaller than SLNR_k^{\max} given in Section II.

On the other hand, the ultimate performance is decided by post-SINR. Clearly, the decoded signal should take the form

$$\hat{\mathbf{s}}_k = \mathbf{G}'_k \mathbf{H}_k \mathbf{F}_k \mathbf{s}_k + \mathbf{G}'_k \left(\mathbf{H}_k \sum_{i=1, i \neq k}^K \mathbf{F}_i \mathbf{s}_i + \mathbf{n}_k \right). \quad (14)$$

Thanks to diagonal form in (7) and (8), the covariance matrix of noise vector is given by $\mathbb{E}(\mathbf{G}'_k \mathbf{n}_k \mathbf{n}_k^H \mathbf{G}'_k^H) = \sigma^2 \mathbf{I}_L \mathbb{E}(\mathbf{G}'_k \mathbf{G}'_k^H) = \gamma^2 \sigma^2 \mathbf{I}_L \text{diag}(\theta_1, \dots, \theta_L)$. Furthermore, it can be verified through numerical results (difficult via theoretical analysis though) that the residual CCI is much smaller than the noise power at high SNR. As such, the SINR on the l -th stream, η'_l can be approximately calculated as $\eta'_l = (\gamma^4 \theta_l^2) / (\gamma^2 \sigma^2 \theta_l) = \gamma^2 \theta_l / \sigma^2$. Then, for any two streams l and m with $l > m$, the margin of $\Delta'_{l,m}$ between η'_l and η'_m in terms of decibel (dB) can be expressed as

$$\Delta'_{l,m} = 10 \log_{10} (\eta'_l / \eta'_m) = 10 \log_{10} (\theta_l / \theta_m). \quad (15)$$

Following the same analysis, the margin of $\Delta_{l,m}$ for the

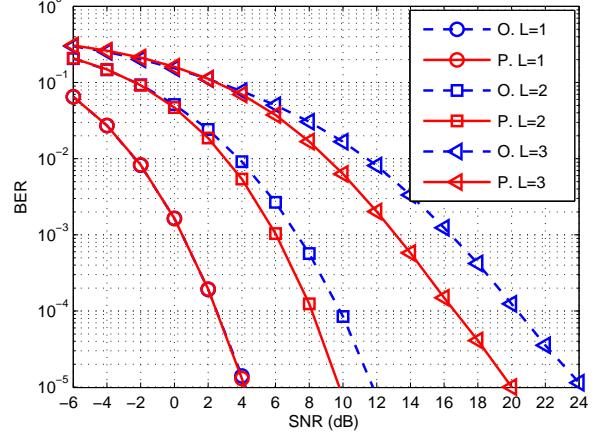


Fig. 1. Uncoded BER of a MU-MIMO system with $N = 8$ transmit antennas at the BS and $K = 2$ users each with $M = 3$ receive antennas.

original scheme can be analogously calculated as

$$\Delta_{l,m} = 10 \log_{10} (\eta_l / \eta_m) = 10 \log_{10} (\lambda_l / \lambda_m). \quad (16)$$

According to (13), we have $\lambda_l / \lambda_m = (\theta_l \omega_m) / (\theta_m \omega_l)$. Further, we have that $\omega_m > \omega_l$ for $l > m$ by definition. It then ensures that the following inequality holds:

$$\Delta'_{l,m} < \Delta_{l,m}. \quad (17)$$

This explicitly shows that the SINR margin between any two streams decreases by applying the proposed scheme. In other words, the effective channel gains between the multiple streams are now less unbalanced. Its effectiveness will be further examined by simulation in the next section.

IV. SIMULATION RESULTS

Fig. 1 compares the simulated bit error rate (BER) per user in a MU-MIMO system with different system configurations. Here, P denotes the proposed precoding scheme and O denotes the original scheme in [3]. QPSK modulation with Gray mapping is employed and the BER curves are plotted versus the transmit SNR (L/σ^2). It is seen that the proposed scheme and the original scheme for single-stream case ($L = 1$) achieve the same BER performance. For multiple streams ($L = 2$ and 3), the former outperforms the latter with sizeable gains. In specific, a gain of around 2 dB and 4 dB can be achieved at $\text{BER}=10^{-4}$ for streams of $L = 2$ and $L = 3$, respectively. We also carried out the achievable sum-rate comparison. It is found that our scheme is almost the same as the original one. The results are omitted due to page limit.

The above simulation results verify the effectiveness of the proposed precoding scheme over the original SLNR based scheme when there are multiple data streams for each user.

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